

2 Algebra and graphs

2.1 Introduction to algebra

Notes and examples

- 1 Know that letters can be used to represent generalised numbers.
- 2 Substitute numbers into expressions and formulas.

2.2 Algebraic manipulation

Notes and examples

- 1 Simplify expressions by collecting like terms.
- 2 Expand products of algebraic expressions.
- 3 Factorise by extracting common factors.
- 4 Factorise expressions of the form:
 - $ax + bx + kay + kby$
 - $a^2x^2 - b^2y^2$
 - $a^2 + 2ab + b^2$
 - $ax^2 + bx + c$
 - $ax^3 + bx^2 + cx$.
- 5 Complete the square for expressions in the form $ax^2 + bx + c$.

Simplify means give the answer in its simplest form,
e.g. $2a^2 + 3ab - 1 + 5a^2 - 9ab + 4 = 7a^2 - 6ab + 3$.

e.g. expand $3x(2x - 4y)$, $(3x + y)(x - 4y)$.
Includes products of more than two brackets,
e.g. expand $(x - 2)(x + 3)(2x + 1)$.

Factorise means factorise fully,
e.g. $9x^2 + 15xy = 3x(3x + 5y)$.

2.3 Algebraic fractions

Notes and examples

- 1 Manipulate algebraic fractions.
- 2 Factorise and simplify rational expressions.

Examples include:

- $\frac{x}{3} + \frac{x-4}{2}$
- $\frac{2x}{3} - \frac{3(x-5)}{2}$
- $\frac{3a}{4} \times \frac{9a}{10}$
- $\frac{3a}{4} \div \frac{9a}{10}$
- $\frac{1}{x-2} + \frac{x+1}{x-3}$.

e.g. $\frac{x^2 - 2x}{x^2 - 5x + 6}$.

2 Algebra and graphs (continued)

2.4 Indices II

Notes and examples

1 Understand and use indices (positive, zero, negative and fractional).

e.g. solve:

- $32^x = 2$
- $5^{x+1} = 25^x$.

2 Understand and use the rules of indices.

e.g. simplify:

- $3x^{-4} \times \frac{2}{3}x^{\frac{1}{2}}$
- $\frac{2}{5}x^{\frac{1}{2}} \div 2x^{-2}$
- $\left(\frac{2x^5}{3}\right)^3$.

Knowledge of logarithms is **not** required.

2.5 Equations

Notes and examples

1 Construct expressions, equations and formulas.

e.g. write an expression for the product of two consecutive even numbers.

Includes constructing simultaneous equations.

2 Solve linear equations in one unknown.

Examples include:

- $3x + 4 = 10$
- $5 - 2x = 3(x + 7)$.

3 Solve fractional equations with numerical and linear algebraic denominators.

Examples include:

- $\frac{x}{2x+1} = 4$
- $\frac{2}{x+2} + \frac{3}{2x-1} = 1$
- $\frac{x}{x+2} = \frac{3}{x-6}$.

4 Solve simultaneous linear equations in two unknowns.

5 Solve quadratic equations by factorisation, completing the square and by use of the quadratic formula.

Includes writing a quadratic expression in completed square form.

Candidates may be expected to give solutions in surd form.

The quadratic formula is given in the List of formulas.

6 Change the subject of formulas.

e.g. change the subject of a formula where:

- the subject appears twice
- there is a power or root of the subject.

2 Algebra and graphs (continued)

2.6 Inequalities

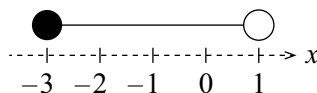
Notes and examples

1 Represent and interpret inequalities, including on a number line.

When representing and interpreting inequalities on a number line:

- open circles should be used to represent strict inequalities ($<$, $>$)
- closed circles should be used to represent inclusive inequalities (\leq , \geq)

e.g. $-3 \leq x < 1$



2 Construct, solve and interpret linear inequalities.

Examples include:

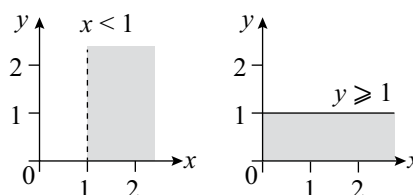
- $3x < 2x + 4$
- $-3 \leq 3x - 2 < 7$.

3 Represent and interpret linear inequalities in two variables graphically.

The following conventions should be used:

- broken lines should be used to represent strict inequalities ($<$, $>$)
- solid lines should be used to represent inclusive inequalities (\leq , \geq)
- shading should be used to represent unwanted regions (unless otherwise directed in the question).

e.g.



4 List inequalities that define a given region.

Linear programming problems are not included.

2.7 Sequences

Notes and examples

1 Continue a given number sequence or pattern.

Subscript notation may be used, e.g. T_n is the n th term of sequence T .

2 Recognise patterns in sequences, including the term-to-term rule, and relationships between different sequences.

Includes linear, quadratic, cubic and exponential sequences and simple combinations of these.

3 Find and use the n th term of sequences.

2 Algebra and graphs (continued)

2.8 Proportion

Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities.

Notes and examples

Includes linear, square, square root, cube and cube root proportion.

Knowledge of proportional symbol (\propto) is required.

2.9 Graphs in practical situations

- 1 Use and interpret graphs in practical situations including travel graphs and conversion graphs.
- 2 Draw graphs from given data.
- 3 Apply the idea of rate of change to simple kinematics involving distance–time and speed–time graphs, acceleration and deceleration.
- 4 Calculate distance travelled as area under a speed–time graph.

Notes and examples

Includes estimation and interpretation of the gradient of a tangent at a point.

Areas will involve linear sections only.

2.10 Graphs of functions

- 1 Construct tables of values, and draw, recognise and interpret graphs for functions of the following forms:
 - ax^n (includes sums of no more than three of these)
 - $ab^x + c$
 where $n = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2, 3$; a and c are rational numbers; and b is a positive integer.
- 2 Solve associated equations graphically, including finding and interpreting roots by graphical methods.
- 3 Draw and interpret graphs representing exponential growth and decay problems.
- 4 Estimate gradients of curves by drawing tangents.

Notes and examples

Examples include:

- $y = x^3 + x - 4$
- $y = 2x + \frac{3}{x^2}$
- $y = \frac{1}{4} \times 2^x$.

e.g. finding the intersection of a line and a curve.

2 Algebra and graphs (continued)

2.11 Sketching curves

Recognise, sketch and interpret graphs of the following functions:

- linear
- quadratic
- cubic
- reciprocal
- exponential.

Notes and examples

Functions will be equivalent to:

- $ax + by = c$
- $y = ax^2 + bx + c$
- $y = ax^3 + b$
- $y = ax^3 + bx^2 + cx$
- $y = \frac{a}{x} + b$
- $y = ar^x + b$

where a , b and c are rational numbers and r is a rational, positive number.

Knowledge of turning points, roots and symmetry is required.

Knowledge of vertical and horizontal asymptotes is required.

Finding turning points of quadratics by completing the square is required.

2.12 Functions

1 Understand functions, domain and range, and use function notation.

2 Understand and find inverse functions $f^{-1}(x)$.

3 Form composite functions as defined by $gf(x) = g(f(x))$.

Notes and examples

Examples include:

- $f(x) = 3x - 5$
- $g(x) = \frac{3(x+4)}{5}$
- $h(x) = 2x^2 + 3$.

e.g. $f(x) = \frac{3}{x+2}$ and $g(x) = (3x+5)^2$. Find $fg(x)$.

Give your answer as a fraction in its simplest form.

Candidates are **not** expected to find the domains and ranges of composite functions.

This topic may include mapping diagrams.